A Linear Prediction Approach to Joint Estimation of Water/Fat Images and Field Inhomogeneity Map

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INTRODUCTION

Three-point water/fat imaging techniques [1,2] allow the determination of separate water and fat images while overcoming the limitations of Dixon's original two-point method arising from B_0 field inhomogeneity. While the determination of the field map is considered secondary for water and fat separation in Dixon imaging, it can be useful in some other applications, *e.g.*, for lipid signal removal and metabolite quantitation in MR spectroscopic imaging. Here we propose a simple and efficient method based on linear prediction for joint estimation of the water and fat images as well as the field inhomogeneity map from a Dixon-type acquisition.

MATERIALS AND METHODS

The signal within a voxel can be modeled as

$$s(n) = \left(\rho_w e^{j\varphi_w} + \rho_l e^{j\varphi_l} e^{j2\pi t_n \Delta f_{wl}}\right) e^{j2\pi t_n \Delta f_B}, \quad n = 0,1,2 \tag{1}$$

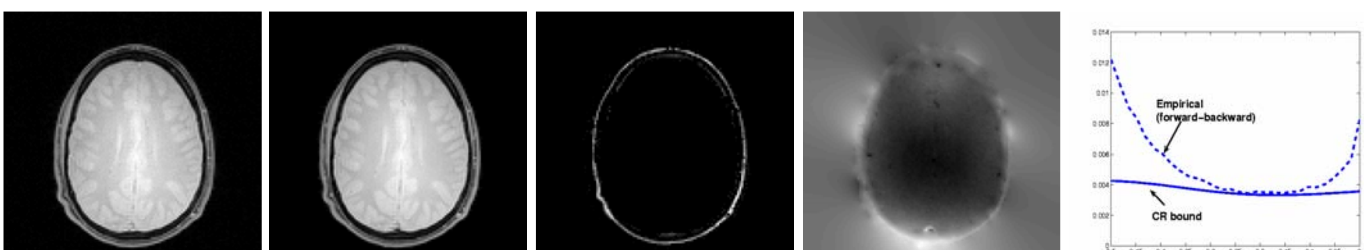
where ρ_w and ρ_l represent the water and lipid components, respectively, Δf_{wl} is the frequency shift between the water and lipid resonances and Δf_B is the frequency shift due to field inhomogeneity. In [3], an iterative method is proposed for estimation of ρ_w, ρ_l and Δf_B assuming Δf_{wl} is known exactly. However, this estimation problem can be solved nicely using a linear prediction (LP) method as long as the acquisitions are uniformly spaced in time (*i.e.*, $t_n = n\Delta t$). More specifically, the signal in (1) satisfies the linear prediction equation $s(n)=a_1 s(n-1)+a_2 s(n-2)$, whose characteristic polynomial $z^2-a_1 z-a_2=0$ has roots $z_1=e^{j2\pi\Delta f_B}$ and $z_2=e^{j2\pi(\Delta f_B+\Delta f_{wl})}$. The LP coefficients a_1 and a_2 can be efficiently and stably estimated voxel by voxel using a forward-backward LP method [4] (assuming negligible T_2 effects, in which case z_1 and z_2 are on the unit circle). Furthermore, the proposed method can easily include estimation of the T_2 parameters for both signal components, although this extension requires acquiring more than 3 images. Note that we do not fix the chemical shift between water and lipids, since 3 data points suffice to estimate both components. If Δf_{wl} is known precisely, it can be enforced when solving for the linear prediction coefficients. The field map at each voxel can be estimated from the largest component in the estimated signal. Since the frequency estimates will be relatively noisy due to the small number of data points used in linear prediction, we can regularize the Δf_B estimate by imposing spatial smoothness:

$$\Delta\hat{f}_B = \arg\min_{\Delta f} \| \Delta f - \Delta\hat{f}_{B,0} \|_W^2 + \lambda \| D\Delta f \|^2 \tag{2}$$

where $\Delta\hat{f}_{B,0}$ is the voxel-by-voxel estimate, D computes finite differences, W is a weighting matrix obtained from the signal amplitude at each voxel in the images and λ is a regularization parameter which can be determined empirically or based on a model of the noise and the field map.

RESULTS AND DISCUSSION

Figures 1(a)-(d) show the water/fat decomposition and estimated field map for a three-point acquisition using the IDEAL parameters [2]. In the noiseless case, LP produces a correct decomposition of the signal. In the presence of significant noise, the estimates produced by the forward-backward LP method are generally not optimal (in the maximum likelihood sense). Thus, we have compared the estimation error using the proposed method with the Cramér-Rao (CR) bound for different echo shifts between water and lipids. Echo shifts uniformly spaced on the unit circle yield the best estimates, as concluded in [2]. Furthermore, the dependence of the MSE error on the initial phase $\Delta\varphi=\varphi_l-\varphi_w$ is shown in Fig. 1(e), where the CR bound and empirical MSE error for estimation of the amplitude are plotted (even though the estimates have a small bias), for *SNR=8*. As can be observed, there is a range of $\Delta\varphi$ values for which the amplitude estimates have a MSE very close to the CR bound, whereas a symmetric acquisition yields an ill-posed problem. Note that the optimum $\Delta\varphi$ for the proposed method is SNR-dependent and should be determined based on an estimate of the SNR.



(a) Original image (b) Water image (c) Fat image (d) Estimated field map (e) Amplitude estimation error

Fig 1. (a)-(d) Results on a brain image; (e) Comparison of theoretical bounds and empirical MSE for amplitude estimation.

CONCLUSION

We have demonstrated the potential of using linear prediction for joint estimation of water/fat images and the field map from Dixon-type acquisitions. The linear prediction framework can handle the T_2 effect, fixed frequency difference between the water and lipid resonances, as well as the presence of multiple resonances. The proposed method should prove useful as a noniterative alternative to or as a way to provide an initial estimate for the IDEAL algorithm [3].

REFERENCES

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