Lattice Sampling of k-Space for Parallel Imaging

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Introduction

Reconstruction of non-Cartesian SENSitivity-Encoded (SENSE) data often involves solving a large system of equations for the amplitudes of the voxel functions used to model the desired image function [1]. While solving this system has been made faster by utilizing Conjugate Gradient (CG) methods and by clever use of the Fast Fourier Transform (FFT) to implement some of the matrix multiplications, the reconstruction is still too slow for routine clinical applications. This paper proposes a class of sampling trajectories in which reconstruction can be made much faster.

Methods

We will define a lattice sampling function as $S(\vec{k})=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(\vec{k}-n\vec{v}_1-m\vec{v}_2-\vec{k}_0)$, whose point spread function is given by $h(\vec{x})=\frac{e^{i2\pi\vec{k}_0\cdot\vec{x}}}{\det[\vec{p}_1\ \vec{p}_2]}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(\vec{x}-n\vec{p}_1-m\vec{p}_2)$. Here, $\vec{v}_1$ and $\vec{v}_2$ are independent sampling basis vectors, and $\vec{p}_1$ and $\vec{p}_2$ are the periodicity vectors such that $[\vec{v}_1\ \vec{v}_2]^{-T}=[\vec{p}_1\ \vec{p}_2]$.

Lattice sampling of k-space produces periodic replicates of the original image on a related lattice in the image domain, with a complex exponential weighting related to the offset of the lattice from the origin of k-space. Aliasing will occur if the lattice is undersampled, but only a small set of image values is aliased onto a single voxel if we neglect the effects of finite sampling. Given data from multiple distinct channels, this aliasing can be unfolded. Cheung [2] uses this formulation to describe a method to use integer-factor decimated lattice samples to reconstruct image values on a grid defined by the periodicity vectors. In the special case when the sampling basis vectors are aligned with the coordinate axes, this also reduces to standard Cartesian SENSE [3]. This paper allows more general lattices to be used. Specifically, invoke the usual SENSE reconstruction equation: $A\vec{\rho}=\vec{g}$, where $\vec{\rho}$ is the vector of N^2 image voxels, $\vec{g}$ is a vector of the LN^2 aliased voxel values obtained from the Fourier transforms of the L data sets evaluated on the $N\times N$ voxel grid, and A is the $LN^2\times N^2$ matrix describing the sensitivity-weighted relationship between each aliased voxel and the true image voxels. A very desirable property of lattice sampling is that A is extremely sparse. In the special case of Cartesian lattices, A becomes a block-diagonal matrix. The sparsity of A can be exploited to significantly speed-up the reconstruction process. There is one caveat, however, since the aliasing image locations will not all necessarily correspond to the voxel grid points we want to reconstruct. In this case, we can approximate each off-grid image point as a weighted combination of its neighboring voxels. This approximation has the effect of spatial regularization (or smoothing), since neighboring pixels are now coupled together in the reconstruction. This property is desirable because A is often ill-conditioned.

This sparse matrix treatment can also be extended to k-space trajectories that are comprised of a composite of lattice patterns. A useful special case is the zig-zag EPI k-space trajectory (Figure 1), which can be treated as a combination of two different lattice structures. By treating each lattice separately, we double the number of data sets but also double the amount of aliasing in each of these "pseudo channels". Because the sampling basis vectors for each of these lattices are different, the corresponding periodicity vectors are also different, so the aliasing image points in the reconstructions from each lattice are different. In addition, since each lattice structure has an offset from the origin of k-space, reconstructions of these lattices will have a linear complex-exponential weighting. This can be absorbed into the sensitivity profiles of the new pseudo channels. Thus, we can form two lattice matrix equations and solve them simultaneously for the image values.

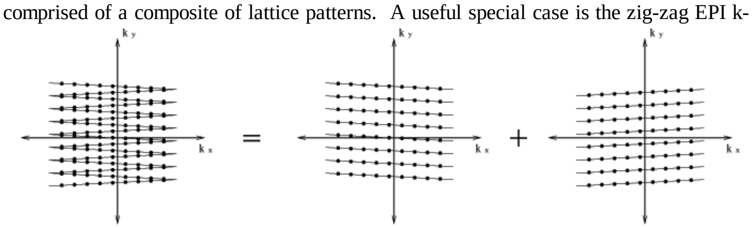

Figure 1. Decomposition of the zig-zag EPI trajectory into two lattices.

Results

The proposed algorithm has been implemented and tested in the MATLAB environment. A set of representative results is shown in Fig. 2, where data was created for 6 receiver coils using a zig-zag EPI trajectory with an undersampling factor of 4 and additive white Gaussian noise. Figure 2(a) shows the reference image used for simulation. Figure 2(b) was reconstructed in 63.8 seconds by directly inverting the sparse matrix, using an approximate minimum degree column ordering on the A matrix and using Cholesky decomposition to solve the resulting normal equations. Figure 2(c) shows an image reconstructed in 410.4 seconds by terminating the intensity-corrected CG algorithm from [1] after 28 iterations. Figure 2(d) was reconstructed in 1034.3 seconds using the intensity-corrected CG algorithm terminated after 70 iterations.

Conclusion

This paper proposes a new class of sampling trajectories for parallel imaging, which admit fast solution of the reconstruction problem. Using the proposed lattice formulation, the paper also successfully solves the SENSE reconstruction problem for zig-zag EPI trajectories, which can have immediate practical impact.

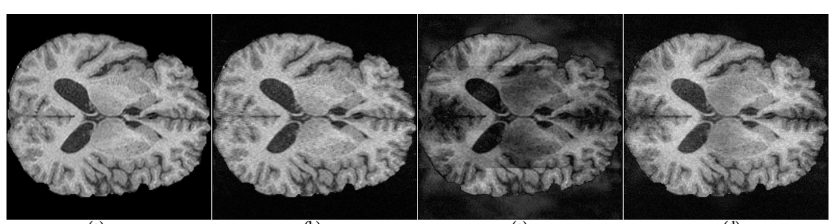

Figure 2. Reconstruction of SENSE data from a zig-zag EPI trajectory. (a) Original image. (b) New algorithm, 63.8 seconds. (c) Pruessmann's algorithm, 28 iterations, 410.4 seconds. (d) Pruessmann's algorithm, 70 iterations, 1034.3 seconds.

References

[1] Pruessmann KP, *et al.* Magn Reson Med 2001; **46**: 638-651.
[2] Cheung KF in *Advanced Topics in Shannon Sampling and Interpolation Theory*. Marks RJ, ed. Springer-Verlag 1993, 86-119.
[3] Pruessmann KP, *et al.* Magn Reson Med 1999; **42**: 952-962.