A New Approach for Optimal Reconstruction Using Rescaled Matrices from Non-uniformly Sampled K-space Data

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Synopsis

K-space gridding is commonly performed following non-uniform K-space MRI data acquisition. A drawback of conventional gridding algorithm is that the image quality depends on the density compensation function (DCF) used in the pre-compensation step. We present a new simple reconstruction algorithm that does not need density compensation. Instead, iterative procedures are performed on a rescaled matrix larger than the original-sized grid; the resulting reconstructed images are of quite high quality. The proposed algorithm provides a new approach for optimal image reconstruction; it represents an alternative to the previously proposed URS/BURS algorithms but with equal or better image quality.

Introduction

It is often difficult to optimize a DCF in the conventional gridding algorithm (1). A new simple reconstruction algorithm that does not require a DCF is presented here.

Methods

The newly proposed algorithm is an iterative algorithm and an extension of 'next neighbor re-gridding algorithm' proposed earlier (2,3). Figure 1 shows a flow chart of the basic 'Iterative Next Neighbor re-Gridding (INNG) algorithm'. Suppose that the original target grid is an N x N matrix. K-space data are first distributed into *s*N x *s*N matrix, where $s = 2^m$ (*m* is a small positive integer) (Fig.1(a)). The location of each datum in the large rescaled matrix is determined by rounding off the original k-space coordinate after multiplying it by the scale factor *s*. If more than one data share the same matrix coordinate, the mean value is stored. An Inverse Fourier Transform (IFT) is performed on the matrix (a), leading to image matrix (b). Zeros are set outside of the central N x N matrix in (b), resulting in (c). A FT is performed on image (c), leading to a new estimate of the rescaled data (d). The original data in the rescaled matrix (a) are returned to their original locations, as shown in (e); other locations are left to estimates obtained from inversion of (c). An IFT is performed on (e). The image appears in the central N x N matrix on (b). The procedures (b) → (c) → (d) → (e) → (b) (surrounded by dashed lines in Fig.1) are repeated until the difference between the updated image and the image at the previous iteration becomes sufficiently small.

In practice, *s* is set to at least 4 to reduce the errors caused by data shifts in the large matrix. However, the speed of covergence is inversely related to *s*. Hence, a second generation INNG, the facilitated INNG algorithm, was developed to improve the computaitonal efficiency. Figure 2 shows the flow chart of the facilitated INNG algorithm. It consists of the basic INNG algorithm with consecutively increasing scaling factors. The basic INNG algorithm with $s = 2$ is first performed, and continued until convergence is achieved. The image reconstructed in this step will be used as a starting image for the basic INNG algorithm with $s = 4$ in the next step. As such, the matrix scaling factor *s* is increased on successive iterations of converged INNG.

To test this algorithm, k-space data were calculated along 10 interleaved spiral trajectories from a numerical phantom (128x128 matrix) (Fig.3). Noisy data were also generated by adding Gaussian white noise to the ideal simulated data (the mean of the noise was 0 and the standard deviation of the noise was 20% of the average magnitude of the ideal data.). Both ideal and noisy data were reconstructed using the basic INNG (s=8 (101iterations)) and facilitated INNG (s=2 (7iterations), 4 (5iterations), 8 (18iterations)) algorithms, the conventional gridding algorithm with Voronoi DCF (4) and the BURS algorithm (5,6). The root mean square (RMS) error was measured for each reconstructed image. The image SNR was measured for each image from the noisy data.

Results

Figure 4 shows the image profiles from the ideal data reconstructed using the facilitated INNG algorithm (the corresponding lines are indicated in Fig.3). Figure 4 shows no observed profile distortions. Table 1 summarizes the RMS errors and the image SNR. Note that the facilitated INNG algorithm has the least RMS error for ideal data. The measured SNR of the facilitated INNG algorithm is equivalent to that of the conventional gridding and higher than that of the BURS algorithm.

Discussion

One of the main advantages of the INNG algorithms is that they lead to high quality reconstructed images in general non-uniform sampling schemes. Furthermore, the algorithms do not require the precalculated DCF. When a large scaling factor is used to reduce the data shift errors in the basic INNG algorithm, the convergence speed becomes slow. The facilitated INNG algorithm has overcome this disadvantage while it produces high quality reconstructed images. It is often difficult to optimize the DCF in the conventional griddding algorithm and to determine regularization parameters in the BURS algorithm. The INNG algorithms do not require such parameter adjustments. The newly proposed algorithms are considered to be a simple new approach for optimal reconstruction from non-uniformly sampled k-space data.

Acknowledgements

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References

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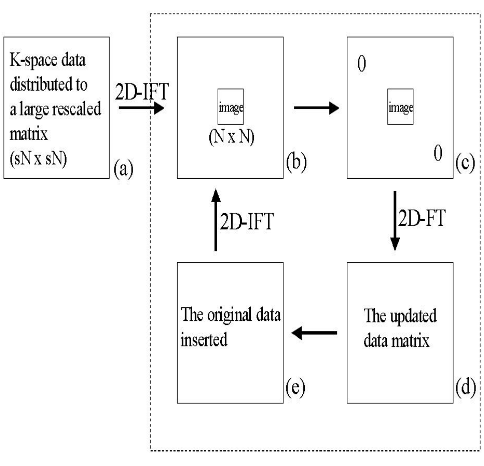

Fig. 1. A flow chart of the basic INNG algorithm

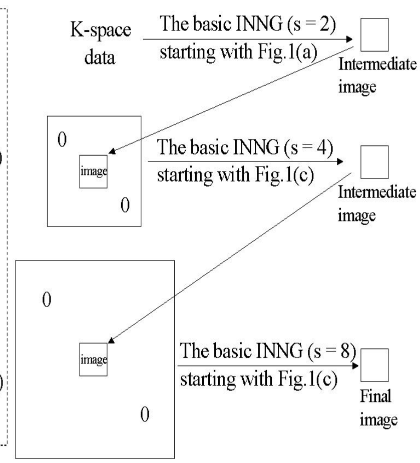

Fig. 2. A flow chart of the facilitated INNG algorithm

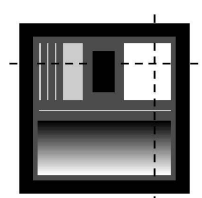

Fig. 3. A phantom

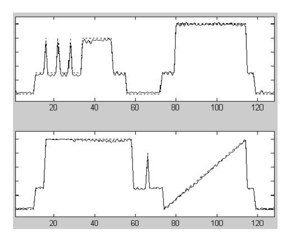

Fig. 4. The profiles of the image using the facilitated INNG algorithm (ideal data)

	Ideal data	Noisy data	
	RMS error	RMS error	Image SNR
Basic INNG (s = 8)	0.02312	0.07381	22.5
Facilitated INNG	0.02162	0.08235	19.1
Conventional gridding	0.02558	0.08942	19.1
BURS	0.02752	0.07796	16.8

Table1. RMS error and SNR comparison