Spatially selective excitations can increase the acquisition efficiency of MRI by encoding the signal prior to acquisition using encoding bases other than the Fourier basis. However, spatially selective excitations require relatively long RF pulses which may limit their usefulness. Spiral trajectories through k-space can be used to minimize excitation time for 3D MRI by optimizing the k-space trajectory. Spiral trajectory use during acquisition (of 2D encoding functions) or in acquisition (of 2D responses) allows us to maintain image quality in 3D MRI, while achieving speedups through software-based adaptive encoding methods.

**Introduction:** Dynamic adaptive MRI achieves imaging time reduction by representing the acquired image using a basis that near-optimally encodes the field of view (FOV) contents [1]. In particular, certain bases decouple the noise component from the MR signal, allowing the basis to be truncated by removing the noise components [2]. In general, k-space can be traversed partially via excitation (when the excitation is used for encoding in conjunction with a gradient) and partially post-excitation (i.e., via gradients before and during acquisition). When low flip angle RF excitations are used with encoding gradients, the resulting excited spatial profile can be expressed as

$$p(x) = \alpha \int \hat{p}(\hat{k}_e(t)) \exp(i\hat{k}_e(t)) d\hat{k}$$

[3], (\alpha \text{ a weighting factor}), where \( \hat{k}_e \) represents the 3D k-space trajectory \( (k_x(t), k_y(t), k_z(t))^T \) visited during the excitation (a vector of continuous functions). After excitation, gradients can be used to traverse the remaining k-space locations (the discrete \( \hat{k}_a \)), leading to the imaging equation

$$S(\hat{k}_e, \hat{k}_a) = \int p(x) \exp(i\hat{k}_e \cdot x) dx$$

for the acquired signal stemming from the spin density \( p(x) \). Using the linear system approach of [4], in the low flip angle regime, we can replace the spatial excitation integral with a summation, by treating it as a set of impulses that instantaneously flip magnetization.

Therefore excitation encoding and post-excitation encoding can be combined:

$$S(\hat{k}_e, \hat{k}_a) = \sum \hat{p}(\hat{k}_e) \int \rho(x) \exp(i\hat{k}_e \cdot x + \hat{k}_a) dx$$

Note that the signal in this case is composed of the complete set of k-space samples (e.g., including all \( \hat{k}_e \) and \( \hat{k}_a \)), weighted by the RF pulse coefficients. Unlike phase encoding (generated by gradients alone), encoding by RF pulses allows arbitrary combinations of the k-space samples via the \( \hat{p}(\hat{k}_e) \). The linear impulse response of the MR system, (equivalent to Fourier phase encoding), can be obtained by repeating the MR experiment, each time selecting a single \( \hat{k}_e \) for which \( \hat{p}(\hat{k}_e) = 1 \), while all others being 0. The specific k-space location \( \hat{k}_e \) corresponding to the \( i \)th impulse of the excitatory RF pulse and produces a digitized signal corresponds to the set of k-space samples at locations \( \hat{k} = \hat{k}_e + \hat{k}_a \), e.g., along the post-excitation trajectory. When arbitrary RF pulses are used, the imaging equation translates into a linear system: the RF coefficient vector \( \hat{p}(\hat{k}_e) \) multiplies the system response matrix (e.g., k-space samples of \( \rho(x) \), \( S(\hat{k}_e, \hat{k}_a) \)). By carefully choosing a set of \( \hat{p} \)'s for the repetitions of the MR experiment, matrix factorization techniques (e.g., [6]) can be applied to approximating the response matrix of the sample. In this work we concentrate on the special cases where either the set of locations \( \hat{k}_e \) or \( \hat{k}_a \) are drawn from a 2D spiral trajectory [5] to optimize traversal of the 3D k-space.

**Methods:** Suppose that either the excitation or the acquisition encoding is performed along a spiral trajectory. In order to fully encode a 3D volume, the other encoding set must span the remaining orthogonal dimension, ensuring that all k-space locations for the given FOV and resolution are accounted for. In the sequence on the left, a 2D spatially selective RF pulse and excite it along the x-y spiral trajectory. For each impulse, in the RF there is a corresponding k-space location \( \hat{k}_e = (\hat{k}_x, \hat{k}_y, 0)^T \). The spatial profile is equally excited along the third dimension (z). A spiral excitation trajectory shortens the RF pulse compared to other 2D k-space traversal methods (e.g., Cartesian echo planar). The response of the profile along z can then be acquired by Fourier encoding the signal along the third orthogonal dimension during readout. Only those \( p(x, y) \) that have maximal response need be used to reconstruct \( \rho(x) \). Since there are only \( N \) locations along z, at most \( N^2 \) 2D profiles are necessary. The system response matrix is tall and narrow; each row of that matrix corresponds to a single choice of \( \hat{k}_e \) and each column corresponds to Each readout sample along \( (0, 0, \hat{k}_z) \). The rank of that matrix is at most \( N \) and can be efficiently encoded with fewer than \( N^2 \) excitations (for an 1-shot spiral) when the set of orthogonal \( p(x, y) \)'s used is truncated [1]. Alternatively, the 2D spiral can be traversed during signal acquisition (pulse sequence on the right). In that case, only \( \hat{k}_z \) needs to be non-Fourier encoded, which is accomplished with a 1D spatial encoding. The system response matrix contains a row for each \( \hat{k}_z \), and columns contain the samples along \( k_x, k_y \) that lie on the spiral. These are the samples produced by the readout process. This matrix, short and wide can also be very efficiently encoded, with fewer than \( N \) excitations.

**Results:** The latter sequence was used to excite 1D spatial basis functions along z and acquire their response along a \( \hat{k}_z \) spiral trajectory. A log plot of the response of the phantom to phase encoding shows that for a typical SNR, almost all of the 128 phase encodes are more significant that noise. Using a near-optimal encoding basis [1] about 30 basis functions encode the entire object. The results of exciting all the basis functions is shown in the figure. As expected from the plot, image information is lost only if truncation exceeds the SNR cutoff (e.g., 16 basis functions, the horizontal bar in the center of the axial slice is blurred).

**Acknowledgments:** This research was supported in part by NIH NCI R01-CA78299, NIH NCI P01-CA67165, a BWH Research Council Interdisciplinary Seed Grant, an Edgerly Science Partnership Award and an Oxygen Alliance grant. Equipment support was provided by Mercury Computer Systems Inc. (Chelmsford, MA).

**References**