Characterising a Population Distribution of Diffusion Tensors

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Abstract: Group analyses of DT-MRI data have so far focused on comparison of scalar variables derived from the tensor, such as the trace or anisotropy. An increasingly popular approach is to spatially normalise such scalar data sets and perform comparisons on a voxel-by-voxel basis. Here, we show how this approach can be extended to comparisons of the whole tensor and suggest statistical measures for characterizing a population distribution of tensors. In particular, we show how to determine the mean, median and mode of a distribution of tensors together with measures of tensor dispersion.

Theory: Pennec and Ayache[1] have shown how the statistical concepts of mean, median and mode are relevant not only to scalar data but also to more complex geometrical data. The key idea is to redefine such measures in a more abstract form. Fréchet[2] defined a continuum of central locations μ_r, ($r \in \Re^+$), where the location μ_r is the element of the domain that minimizes the sum of the distances, raised to the power r, to the samples, e.g. μ_2 is the closest to the samples in a distance-squared sense. Griffin[3] has shown that, for scalar variables, the Fréchet-defined μ_2 is the mean, μ_1 is the median and (with appropriate taking of limits), μ_0 represents the mode(s). To apply this approach to tensor data, a distance metric, d, is required for which we propose the measure:

$$d(\mathbf{A},\mathbf{B}) := \sqrt{(\mathbf{A}-\mathbf{B}):(\mathbf{A}-\mathbf{B})}\,.$$

Using this metric, it can be shown that μ_2 coincides with the obvious definition of the mean of a set of tensors i.e.

$$\mu_2(\{\mathbf{D}_1,\ldots,\mathbf{D}_n\}) = \frac{1}{n}\sum_{i=1}^{n}\mathbf{D}_i$$

The median tensor is computed by starting with the mean tensor and gradient descending on the absolute distance function. The mode is determined by starting with median tensor and repeatedly gradient descending on the r-distance function. For the first gradient descent, r is set to 0.9, for the second 0.8 and so on. At the completion of gradient descent with $r = 0.1$, the sample closest to $\mu_{0.1}$ is selected as the mode tensor. This approach also provides a means for generating dispersion measures:

$$s_r = \left(\frac{1}{n}\sum_{i=1}^{n}\left|d(\mu_r, D_i)\right|^r\right)^{\frac{1}{r}}$$

Of most interest are S_2 and S_1, which are the standard deviation and mean absolute deviation respectively. Dividing by the magnitude of the mean and median gives the normalized deviations, e.g. $\overline{S_2} = S_2 / \|\langle\mathbf{D}\rangle_2\|$ and $\overline{S_1} = S_1 / \|\langle\mathbf{D}\rangle_1\|$

Application: DT-MRI data, with isotropic resolution (2.5 × 2.5 × 2.5mm), were collected from 60 slice locations from eleven healthy male volunteers (age = 33.3 ± 4.7 years). To compare tensors in a voxel, it was first necessary to co-register the DT-MRI data sets. Since rotation and shear of the image volume reorients the tensors in each voxel, the tensor reorientation strategy proposed by Alexander et al.[4] was used to correctly reorient each tensor during the registration procedure. The mean, $\langle D_2\rangle$, median, $\langle D_1\rangle$, and mode, $\langle D_0\rangle$, tensors were then computed in each voxel, together with dispersion measures $\overline{S_2}$ and $\overline{S_1}$, and rotationally invariant indices such as the trace and fractional anisotropy.

References:
1. Pennec X, Ayache N. *J Math Imag Vis* 1998; 9: 49
2. Frechet M. *Annales de l'Institut Henri Poincaré* 1948; X: 215.
3. Griffin LD. *Image and Vision Computing* 1997; 15: 369-398.
4. Alexander A, Gee J, Bajcsy R. *Proc MICCAI* 1999

Results:

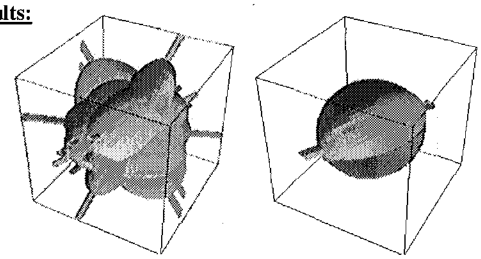

Figure 1: Example of averaging of 10 tensors in a voxel. In the left panel, each tensor is represented as an ellipsoid, and the principal eigenvector with a bar. The right hand panel shows the mean (darker) and median (lighter) tensors.

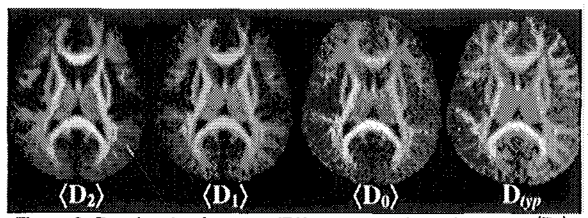

Figure 2: Fractional anisotropy (FA) computed from the mean, $\langle \mathbf{D}_2\rangle$, median, $\langle \mathbf{D}_1\rangle$, and mode, $\langle \mathbf{D}_0\rangle$, of 10 DT-MRI data sets, together with the FA computed from a typical data set ($\mathbf{D}_{typ}$).

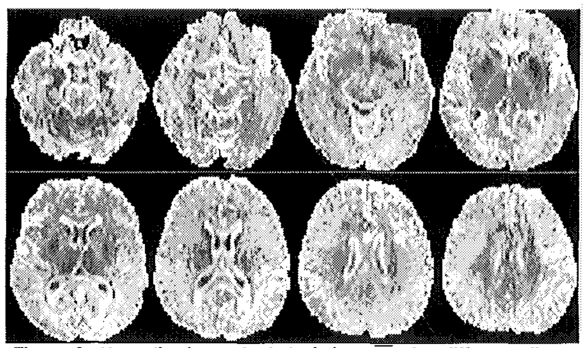

Figure 3: Normalised standard deviation, $\overline{S_2}$, for different slices. Note the image is brighter at tissue interfaces and is darker within central structures.

Discussion: The images computed from the mean and median tensors appeared quite similar for this data set. However, the advantages of using the median of a distribution in the presence of statistical outliers are well known. Images computed from the mode had a grainy appearance, compared to the mean and median images. The mode is only likely to be a useful measure in a much larger number of subjects than used here. However, we include its derivation here for the sake of completeness. An image of S_2 could be a useful tool in studies that use ROI analyses. Low intensity indicates low inter-subject scatter of tensors and helps to identify regions that are less affected by partial volume of different tissue types. Placement of ROIs could therefore be guided by consulting an image of S_2

Conclusion: We have outlined a group of statistical measures for characterizing the distribution of a set of tensors. Analyses of the whole tensor may offer a more complete means of characterizing distributions, and use of the tensor dispersion measures should help to improve the robustness of region-of-interest analyses.