

# A simple model to study acoustic noise induced by transverse gradient coils

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**Abstract:** This work presents a simple model to understand acoustic noise induced by vibrations of MRI transverse gradient coils. The flexible string approximation is used to simulate the vibrations of transverse gradient coils with cylindrical geometry. Forced damped oscillations of finite-length gradient coil supports are studied as a function of the current distribution in the coils, and the spectral density of MRI gradient pulses. The main differences between vibrations of coil support and gradient pulses are related to the free vibration frequencies of coil support. These correspond to the normal modes of coil vibration and depend on size, density and Young's modulus of the coil support.

## INTRODUCTION

A lot of concern about comfort and safety of patients has been raised in the last years due to the high level of acoustic noise induced by MRI pulse sequences [1]. Mansfield et al. introduced the principles of acoustic screening in 1995 to design quiet gradient coils by Lorentz force balancing [2,3]. Nevertheless, these kinds of coils have not been extensively used because of their low gradient and shielding efficiency.

Measurements of the acoustic response of gradient coils carried out in the last years indicate that the noise spectrum is dominated by the gradient pulse spectrum [1], but includes acoustic resonances [4].

In this work, a simple model based on the forced damped oscillations of a flexible string is presented to understand the vibration motion of non-force balanced transverse gradient coils.

## METHOD

For transverse gradient coils with cylindrical geometry, the magnetic pressure applied on the elementary area  $ad\phi dz$  of the gradient coil support is proportional to the strength of the uniform static magnetic field,  $B_0$ , and the  $\phi$ -component of the current density,  $j_\phi(z, \phi) = j(z) \cos\phi$ , flowing at position  $(z, \phi)$ . In order to approximate the gradient coil support by a string, all contributions along  $\phi$  must be integrated, and as a result, the magnetic force applied at  $z = \xi$  is radial and can be written as

$$F(z) dz = 2aB_0 j_0(z) \delta(z - \xi) dz, \quad (1)$$

where  $a$  is the coil radius. The vibrations of the string acted on by a simple harmonic force of angular frequency  $\omega$  applied along the  $x$ -axis at  $z = \xi$  is given by the differential equation

$$\rho \frac{\partial^2 x}{\partial t^2} + \eta \frac{\partial x}{\partial t} - E \frac{\partial^2 x}{\partial z^2} = F(z) e^{-i\omega t}, \quad (2)$$

where  $\rho$ ,  $\eta$ , and  $E$  are the density, viscosity and Young's modulus of the material used to hold the wires.

For a string with length,  $l$ , and boundary conditions  $x = 0$  at  $z = 0$  and  $z = l$ , the steady-state motion can be expressed in terms of a Fourier series

$$x(z, t) = \sum_{n=1}^{\infty} \alpha_n \sin\left(\frac{n\pi z}{l}\right) e^{-i\omega t}. \quad (3)$$

Considering that the magnetic force is distributed along the  $z$ -axis, the general solution of Eqs. (2) and (3) is a linear combination of waves that travel along the string with velocity,  $c = \sqrt{E/\rho}$ , and can be written as:

$$x(z, t) = -\frac{4aB_0}{l\rho} \sum_{n=1}^{\infty} \frac{F_n \sin(n\pi z/l)}{\sqrt{(\omega^2 - \omega_n^2)^2 + K^2 \omega^2}} e^{-i\omega t + \beta} \quad (4)$$

where the form factor of order  $n$ ,

$$F_n = \int_0^l \sin(n\pi z'/l) j_0(z') dz', \quad (5)$$

depends on the current density in the coil. Here, the angular frequencies of free vibration modes,  $\omega_n$ , the effective frictional resistance per unit length,  $K$ , and the phase shift angle,  $\beta$ , are given by

$$\omega_n^2 = \left(\frac{n\pi c}{l}\right)^2 - \left(\frac{\eta}{\rho}\right)^2, \quad K = \frac{\eta}{\rho}, \quad \tan \beta = \frac{\omega K}{\omega^2 - \omega_n^2}. \quad (6)$$

## RESULTS

The time-dependent amplitude of coil support vibrations at  $z = 0$  was calculated using Eq. (4) for different gradient coils designed by the fast-simulated annealing method [5].

Coil vibrations under a broad range of gradient waveforms, including square, trapezoidal, and triangular shapes, were studied in order to mimic gradient pulse sequences. The spectral density of both gradient pulses and vibrations were calculated by using the fast Fourier transform.

Symmetrical gradient coils have shown a resonant behavior characterized by a set of free vibration frequencies  $\nu_n = (n + 1/2) \nu_0$ , with  $\nu_0 = nc/2l$ , when pulses of current with uniform spectral density ("sinc" pulses) were set in the coils. For other waveforms the spectrum of vibrations resembles the gradient spectrum, but the resonance phenomena modulates its amplitude.

## CONCLUSIONS

A simple string model is proposed to study the vibrations of transverse gradient coils with cylindrical geometry. Coil support vibrations were studied for different gradient pulse sequences. Similar spectral densities were found for gradient pulses and coil vibrations. The main differences between spectra are related to the free vibration frequencies. The amplitude of free vibration modes depends on coil size, speed of sound in the coil support and on current distribution.

## REFERENCES

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