An Algorithm for Preservation of Orientation during Non-Rigid Warps of Diffusion Tensor Magnetic Resonance (DT-MR) Images

Daniel ALEXANDER[1], Carlo PIERPAOLI[2], Peter BASSER[3], James C. GEE[4]

[1]UCL (University College London), Computer Science, Gower St.,, London, UK; [2]NB/NINDS/NIH, Building 10-4N252, Bethesda, MD ; [3]National Institutes of Health, Building 13, Room 3W16, Bethesda, MD ; [4]University of Pennsylvania, School of Medicine, Department of Radiology, Philadelphia, Pennsylvania United States;

Introduction

Spatial transformations of images are essential for common post-processing operations such as registration, which is used to normalise collections of data sets into a common spatial reference frame. For scalar images, the application of spatial transformations is straightforward, since the data value at each point in the transformed image can be interpolated directly at the corresponding location in the original image. Transformations of tensor images, however, are more challenging, because the data has an intrinsic orientation with respect to the anatomical structure of the image, which must be handled appropriately.

An algorithm, called the *preservation of principal directions* (PPD), is presented that computes the appropriate reorientation of each diffusion tensor (DT) in a DT-MR image, [1], undergoing a non-rigid transformation. We also present results using synthetic data to validate this method. More details can be found in [2].

Methods

In a DT-MR image, the shape and orientation of the measured DTs reflects the microstructure of the tissue being imaged. If we apply, for example, a rigid rotation, R, to such an image, we must apply a similar rotation to each DT to ensure that the microstructural information it contains is preserved. For the rigid rotation case, this rotation is simple to achieve by a similarity transform of the DT matrix, D, through R: $D \to R^T D R$, which preserves the size and shape of the DT, but rotates its axes through R.

An affine transformation is represented by a linear transformation matrix, F. If we use F directly to compute the DT in the warped image, i.e., $D \to F^T D F$, the eigenvalues of the DT change. This is not desirable. Although we expect the shape of regions of tissue to change under the transformation, we expect the underlying microstructure of the tissue to remain unchanged. Thus, we require a rigid rotation, R, at each point in the image that reflects the amount of reorientation due to F, which we can apply to the DT at that point in order to change its orientation while preserving its shape.

One possible solution to this problem is to decompose F into its rigid rotation, R, and pure deformation, U, components, which are given by F = RU, for any linear transformation matrix F, [3]. R can then be used to reorient each DT in the image as above. This approach is referred to as the *finite strain* (FS) reorientation strategy, [2]. A problem with FS is that there are additional reorientational effects due to U, which depend upon the original orientation of the image/tissue structure. This problem is illustrated in Figure1 where two images with different orientational structure undergo the same deforming transformation: a horizontal shear. The orientation of the horizontal structure is unaffected by the transformation, but that of the vertical structure is changed considerably. FS clearly does not treat this case correctly, since it provides a single, constant R for this affine transformation.

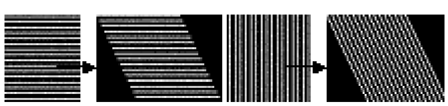

Figure 1. The effect of horizontal shear on anisotropic structures with different orientations.

To find R in the PPD algorithm, we obtain $\underline{e}_1$, $\underline{e}_2$, $\underline{e}_3$, the three eigenvectors of the DT, and $l_1 > l_2 > l_3$, the associated eigenvalues. We define $\underline{n}_i = F\underline{e}_i / |F\underline{e}_i|$, which is the renormalised image of the i-th eigenvector under F. Note that the $\underline{n}_i$ are non-orthogonal.

Where the diffusion profile is prolate ($l_1 >> l_2 =\sim l_3$), the orientation of the tissue microstructure is characterised by $\underline{e}_1$. The tissue orientation after the transformation can be found by applying F directly to $\underline{e}_1$ and normalising the result to obtain $\underline{n}_1$. So we require a rotation to apply to the DT that maps $\underline{e}_1$ to $\underline{n}_1$. Where the diffusion profile is oblate ($l_1 =\sim l_2 >> l_3$), the plane of tissue structure is characterised by $\underline{e}_1$ together with the second eigenvector $\underline{e}_2$. The affine transformation maps this plane to a new plane containing $\underline{n}_1$ and $\underline{n}_2$. So, in this case, we require a rotation for the DT that ensures that the new $\underline{e}_1$ and $\underline{e}_2$ span the same plane as $\underline{n}_1$ and $\underline{n}_2$. In fact, we can take care of both these requirements in a single rotation, R. PPD computes this R separately at each voxel and uses it to reorient only the DT in that voxel. R is the unique rotation matrix that maps $\underline{e}_1$ to $\underline{n}_1$ and $\underline{e}_2$ to a unit vector perpendicular to $\underline{n}_1$ in the plane spanned by $\underline{n}_1$ and $\underline{n}_2$. The PPD method preserves the principal direction of the DT through the transformation as well as the plane of the first two eigenvectors. Therefore, it applies for both prolate and oblate DTs as well as for intermediate DTs for which $l_1 > l_2 > l_3$.

This method is easy to generalise for higher order transformations. We can describe a higher order transformation by a displacement field, $u(\underline{x})$, then replace F by a local linear model derived from the Jacobian, $J_u(\underline{x})$, of u: $F(\underline{x}) = I + J_u(\underline{x})$ and compute R as before.

Experiments and Results

Three warping algorithms for DT-MR images have been tested. The first is a control, which uses no reorientation (NR), the second uses FS, and the third PPD. Results are obtained using synthetic DT data sets for which the gold standard target images are known for a given F, [2].

For a number of affine transformations, the orientation of $\underline{e}_1$ and $\underline{e}_3$ in corresponding voxel locations were compared between transformed images, obtained from each reorientation strategy, and the gold standard. The angles between these eigenvectors are averaged over different regions of the synthetic images, which contain distinct types of DT corresponding to prolate, oblate and spherical diffusion ellipsoids. Figure 2 shows that the PPD driven warping algorithm is the only one for which the difference in the well-defined eigenvector ($\underline{e}_1$ in prolate regions and $\underline{e}_3$ in oblate regions) is close to zero, as it should be.

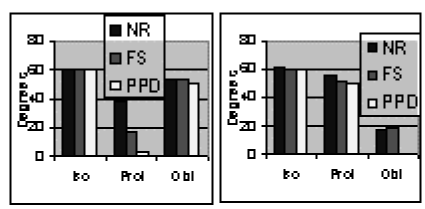

Figure 2 averaged angles between $\underline{e}_1$'s (left) and $\underline{e}_3$'s (right) in regions with different diffusion characteristics.

Discussion

We have described the preservation of principal directions (PPD) algorithm to transform each DT in a DT-MR image under a non-rigid transformation. The development of such an algorithm is a prerequisite for registration of ensembles of DT-MR images. Results shown validate the algorithm over synthetic data and demonstrate its superiority to more naive strategies. Some similar results obtained from intra-subject human data are presented in [2], but the advantages of PPD over FS are not apparent there as in these experiments, because the transformations calculated for these clinical data sets are rigid.

References

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