An Iterative Regularized Algorithm for Tensor Tomography in MRI

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Introduction

One of the approaches used to reduce the effects of motion artifacts in diffusion-weighted imaging (DWI) is to use a projection reconstruction (PR) technique [1]. The PR signal can be presented as a "projection" $p_{ww}(t,q)$ at the projection angle q (defines a readout direction) and spatial position t, which is a function of the spin density function r, diffusion tensor field D, and directional diffusion gradient w. The goal behind DWI is the reconstruction of D, assuming that r is a known function. The conventional PR technique performs measurements for a stationary diffusion gradient (i.e. w is a constant vector). The same diffusion-weighted function is projected at each angle q. Here we propose instead to use rotating diffusion gradients w that are a function of q. We refer to this approach as tensor tomography [2] for which a special iterative reconstruction technique is required [3].

The advantage of using rotating diffusion vectors, as opposed to stationary vectors, is that the acquisition of tensor components is averaged over all space directions and does not contain correlated systematic errors. In particular, this acquisition method may be helpful for countering eddy current effects. Another advantage of tensor tomography is that it has a direct relationship to the solenoidal and irrotational decomposition of the tensor field [4]. In [2,4] the reconstruction of a tensor field was considered where the exponential factor containing D was expanded up to the linear term. The rotating vectors were used to decompose the tensor field into solenoidal and irrotational components. It was shown [4] that a totally solenoidal tensor field can be reconstructed with fewer measurements in comparison to conventional DWI methods.

However, the linear approximation of the diffusion weighted signal does not well represent the projection measurements of a tensor field. Therefore, an iterative approach [3] is necessary to reconstruct the diffusion tensor field from the nonlinear projection measurements. Here is presented an iterative regularized algorithm for the reconstruction of tensor fields in MRI.

Methods

A computer simulation was performed to evaluate our approach. A computer generated phantom was used to simulate the diffusion tensor field that might be expected in a cardiac study. The phantom was comprised of a circular cylindrical tube. The phantom simulated the mid-ventricular wall of the left ventricle. The spin density r; was assumed to be uniform inside the phantom and zero outside. The fiber structure of the myocardium was assumed to be helical with a circumferential fiber angle that varied continuously and linearly from an endocardial position of 60 degrees to an epicardial position of -60 degrees. The diffusion tensor was defined relative to the helical fiber structure with eigenvalues of l_1=1.6, l_2=0.7, and l_3=0.3 ($10^{-3}mm^2s^{-1}$). Since the phantom was independent of the axial coordinate, the simulation considered only a 32x32 slice of the cylinder with an inner radius R_1=7 and an outer radius R_2=14.

Our interest was to determine the fiber direction which meant estimating the first principal direction of the tensor field. This was essentially a 2D problem which involved reconstructing slice by slice a symmetric 3x3 diffusion tensor field D using a slice selective data acquisition. A set of vectors: q =[cos(q), sin(q), 0], a=[-sin(q), cos(q), 0], and b = [0, 0,1] were used to obtain six linear independent projection measurements of the simulated tensor field.

To estimate D, the least squares differences between the model and the measured projections was minimized:

$$L = \sum_{q,t} \sum_{\hat{w}} \left\| p_{\hat{w}\hat{w}}^{model}(D) - p_{\hat{w}\hat{w}}^{measured} \right\|^2 . \qquad (1)$$

In order to minimize (1) with respect to D, a gradient descent (GD) algorithm was used. Since the problem is ill-posed, the reconstruction from noisy data becomes very noisy as L approaches its minimal value at relatively large iteration numbers. It also results in high bias in eigenvalues because of the nonlinear model. Therefore, the following regularization term was added to the expression in (1):

$$L_{reg} = g^2 \sum_{\hat{x}} \sum_{i=1}^{3} (l_i(D) - l_i^{prior}) \quad , \qquad (2)$$

where g^2 was the regularization parameter, and l^{prior} were a priori known eigenvalues of the diffusion tensor field corresponding to heart tissue. The regularization term allows to achieve convergence of the algorithm. It also makes the diffusion tensor a positive-define matrix. Eigenvalues were calculated from the components of the 3x3 diffusion tensor field using analytical functions which were solutions of a cubic equation.

Results

Three reconstructions were performed: from noise-free data, from 10% noisy data without regularization, from 10% noisy data with regularization. All reconstructions were obtained using the same relaxation parameter. Figure 1 presents the behavior of the objective function in the case of reconstructions from noisy data. If regularization is implemented, the algorithm converges after a relatively small number of iterations. Table 1 presents figure of merits for these reconstructions after 3000 iterations. The difference in the angle df between the first principal vector of the phantom and the reconstructed diffusion tensor and the difference dl between the first eigenvalue of the reconstruction and the eigenvalue of phantom were averaged over the region of the heart. According to the values in the table, the nonregularized algorithm provided highly biased eigenvalues and high values for the angular differences. Angle differences were greatly reduced when regularization was applied. Figure 2 provides visualization of the vector field of the reconstruction of the first principal component from noise-free data and the reconstruction from noisy data with regularization.

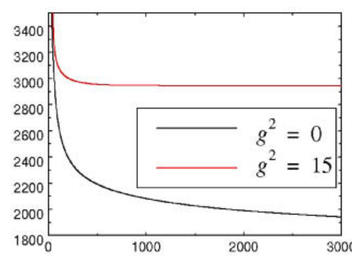

Table 1: Quantitative analysis of reconstructions after 3000 iterations.

	g^2=0	g^2=15
df	42° ±? 28°	22° ±? 16°
dl/l	63% ±? 54%	4% ±? 3%

Figure 1 The objective function as a function of the iteration number. Reconstruction from noisy data.

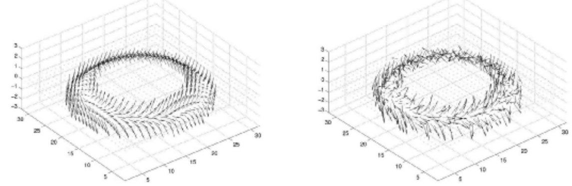

Figure 2 First principal vector. Reconstruction of a tensor field from noise-free projections, 1000 iterations on the left; from noisy (10%) projections, 3000 iterations, g^2=15 on the right.

Discussion

The iterative regularized algorithm works well to reconstruct diffusion tensor fields from tensor tomographic projections. The technique is presently being evaluated using MRI data from excised hearts in experimental animals.

References

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