An Optimal and Efficient New Gridding Algorithm Using Estimation Theory

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Introduction. When sampling under time-varying gradients data is acquired over a non-equally spaced grid in k-space. Gridding algorithms [1-3] are used to interpolate the data onto a Cartesian grid prior to Fourier transform.

A new gridding method is presented here which is both optimal and very efficient. In contrast to conventional gridding, the new method takes into account the presence of measurement noise and allows a tradeoff of SNR against resolution.

Method. Efficient gridding algorithms [1-3] interpolate each uniform output point $f(k_i)$ using a small number of measurements $f(\kappa_j)$ from the neighborhood of this uniform output point. This process is then repeated for each uniform point prior to the Fourier transform. The measurements are accompanied by noise, a fact that is ignored by most gridding algorithms.

Estimation theory [4] lends itself readily to the solution of this problem. Essentially, the problem can be formulated as one of <u>estimating</u> the value of f at k_i using the given set of measurements from the neighborhood of the point. Estimation theory provides a large arsenal of algorithms that can be employed for the solution.

A convenient solution is obtained, if the problem can be described by a linear model. It has already been shown [3] that the regridding solution can be formulated as

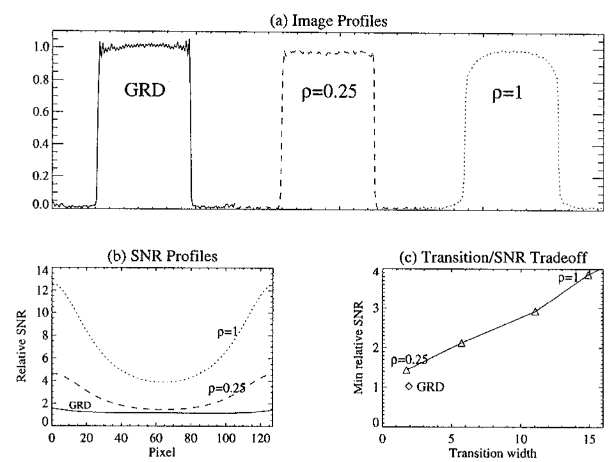
$$\overline{A}\overline{x} = \overline{b}$$

where $\overline{b}$ is the vector of non-uniform measurements, $\overline{x}$ is a vector of uniform points in the neighborhood of k_i, and $\overline{A}$ is an interpolation matrix with $\left[\overline{A}\right]_{ji} = \mathrm{sinc}(|\kappa_j - k_i|)$. Using the linear minimum mean square error estimator, and making some general assumptions regarding the first and second moments of the data, the following solution is obtained:

$$\overline{x} = \overline{A}^T\left(\overline{A}\overline{A}^T + \rho I\right)^{-1}\overline{b} \equiv W\overline{b}$$

This solution is the well-known *Wiener filter*, with $\rho = 1/\mathrm{SNR}$, SNR being the signal-to-noise ratio. The interpolation coefficients are obtained by isolating the row in W corresponding to k_i.

Results. Conventional gridding was compared with the new method using a numerical phantom which was sampled with a four-interleave spiral scan. The phantom consists of a square object occupying half the field of view. A profile through the center of the resulting image is shown in Fig. 1a for conventional gridding and for two values of the parameter ρ. The variance of the noise in the image was also calculated, and the resulting SNR profiles are shown in Fig. 1b. Evidently, the parameter ρ can be used to trade off SNR against resolution; this tradeoff is graphed explicitly in Fig. 1c. For a similar resolution, the new method exhibits better SNR (~40% higher for the example depicted). Moreover, the new algorithm is much more computationally efficient than conventional gridding because it requires fewer interpolation coefficients, and the FFT is performed on an $N \times N$ matrix rather than a $2N \times 2N$ one [3].



Conclusion. A new and optimal gridding algorithm was presented that is more computationally efficient than the conventional method, and allows a tradeoff of SNR against resolution.

References:

[1] Jackson JI, *et al*, IEEE TMI **10**, 473 (1991)
[2] Meyer CH, *et al*, MRM **28**, 202 (1992)
[3] Rosenfeld D, MRM **40**, 14 (1998)
[4] Kay SM, Estimation Theory, Prentice Hall, 1993