

# Analysis and Reduction of the Transient Response in SSFP Imaging

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**Introduction:** Fully refocused steady-state free precession (SSFP) is a fast imaging sequence offering high SNR efficiency. Extensions to SSFP such as fluctuating equilibrium [1] and linear combination SSFP [2] enable water-fat separation and immunity to off-resonance. Because of the volatile transient response during the approach to steady state, data acquisition is usually delayed until this transient dies out. This lengthens the scan time and complicates the ability to perform magnetization-prepared SSFP imaging [3].

In this work, we characterize the transient response in an SSFP sequence. This characterization provides valuable guidance for designing a variable tip-angle excitation sequence for suppressing the undesired transient component.

**Method:** We employ a discrete-time systems framework to analyze the behavior of an SSFP sequence. Let  $x(n)$  represent the excitation input to the spin system where  $n$  is the integer-valued excitation index. Let  $y(n)$  be the complex-valued transverse output from the  $n$ th excitation. If  $x(n) = \alpha\delta(n)$ , where  $\alpha$  is the tip angle and  $\delta(n)$  is the Kronecker delta, then  $y(n) = h(n)$ , the system impulse response. In an SSFP sequence however, we consider  $x(n) = \alpha u(n)$ , where  $u(n)$  is the step function.

Figure 1 gives an SSFP spectral response with off-resonance species  $a, b, c$  labeled for reference. Figure 2 (top row) shows plots of  $|y(n)|$  for off-resonance species  $a, b, c$ , displaying the undesired transient responses which decay to leave the steady-state responses. We characterize this behavior by examining  $Y(z)$ , the  $z$ -transform of  $y(n)$ , because the poles of  $Y(z)$  in the complex  $z$ -plane give the modes of the response. Interestingly,  $Y(z)$  is accurately modeled with a 4th-order system representation, with 3 of the 4 poles dominating the behavior. For example, the dominant poles for species  $b$  are at  $z = 1, 0.99$ , and  $0.98 e^{-i0.97\frac{\pi}{2}}$ . The pole at  $z = 1$  provides the steady-state (or forced) response and is identical to the pole of  $X(z)$ , the  $z$ -transform of the step function input. The pole at  $0.99$  gives a slow decay that depends on the relaxation time constants and tip angle. The pole at  $0.98 e^{-i0.97\frac{\pi}{2}}$ , which corresponds to a  $0.98^n e^{-i(0.97\frac{\pi}{2})n}$  term in the time domain, gives the decaying oscillatory component. This pole turns out to be nearly the same as the pole of  $H(z)$ , the  $z$ -transform of the impulse response  $h(n)$  for species  $b$ . For other off-resonance species, the only significant difference in pole locations is the angle of this complex-valued pole. This angle, which indicates the oscillation frequency, grows with greater amounts of offset frequency.

If the system were linear and time-invariant (LTI), then  $Y(z) = H(z)X(z)$  and the poles and zeroes of the input and impulse response would also appear in the output. Surprisingly, this is nearly the case except for the extra pole at  $0.99$ ; however, this pole gives rise to just a simple decay term. Hence, one can use the intuition of LTI systems analysis to design an  $x(n)$  that suppresses the oscillatory transients. In general,  $x(n)$  should be smoothly varying with primarily low frequency content. From another perspective, one can select  $x(n)$  with an  $X(z)$  containing zeroes near the undesired poles (i.e., those of the oscillatory transients). The challenge is to suppress the transients for all off-resonance species by simultaneously suppressing poles residing nearly all around the unit circle.

**Results:** As inspired by [3], transients can be reduced by halving the tip angle of the initial two excitations. Using our framework, this  $x(n)$  gives an  $X(z)$  with zeroes near  $z = e^{\pm i\frac{\pi}{2}}$ . As seen in Fig. 2 (middle row), this input suppresses transients but over a relatively narrow range of off-resonance species; that is, those with poles near  $z = e^{\pm i\frac{\pi}{2}}$  (e.g., species  $b$ ).

Transient suppression over a wider range of off-resonant frequencies can be achieved by altering the tip angle of additional excitations at the beginning. For example, a 14-point linear ramp excitation sequence produces  $y(n)$  as shown in the bottom row of Fig. 2. The zeroes of this  $X(z)$  are distributed over a greater angular range about the unit circle. Note that transient oscillations in both magnitude and phase are suppressed.

**Conclusion:** The transient behavior of an SSFP sequence is well-characterized by a discrete-time systems framework using a low-order system model. This insight can be used to design an excitation sequence to suppress the oscillatory component of the transients, thereby shortening the preparation period before data acquisition.

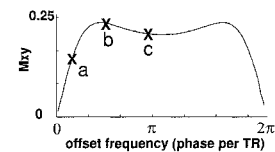


Figure 1: Steady-State Response as a Function of Off-Resonance:  $TR/T1 = 1/200$ ;  $TR/T2 = 1/40$ ;  $\alpha = 30^\circ$ .

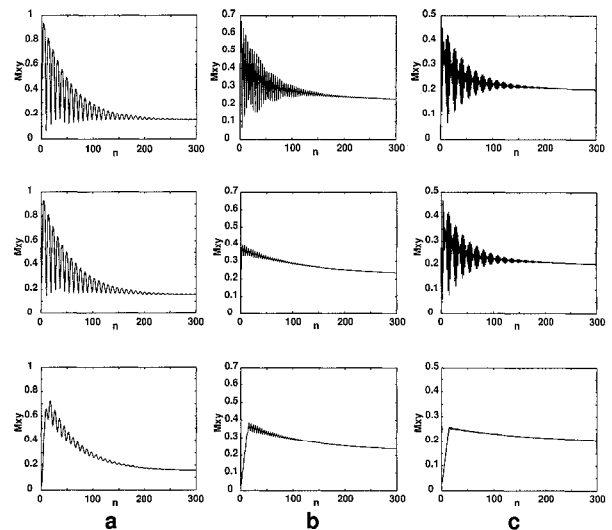


Figure 2: Each row is a plot of  $|y(n)|$  for 3 off-resonance species:  $a, b, c$  as labeled in Fig. 1. Top row: input  $x(n) = \alpha u(n)$ . Middle row:  $x(n)$  with first 2 excitations at  $\alpha/2$ . Bottom row:  $x(n)$  with 14-pt ramp-up at the beginning.

## References:

- [1] VASANAWALA, S. et al., *MRM*, 42:876-883, 1999.
- [2] VASANAWALA, S. et al., *Proc., 7th ISMRM.*, 1:11, 1999.
- [3] DEIMLING, M et al., *Proc., 2nd SMR.*, 1:495, 1994.