Analytical model of susceptibility induced MR signal dephasing by small spherical particles

V.G. Kiselev[1,2] and D.S. Novikov[2,3]

[1] Institute of Medicine, Research Center Jülich GmbH, D-52425 Jülich, Germany; [2] Institute of Physics, Skaryna avenue 70, 220602 Minsk, Byelorussia; [3] California Institute of Technology, Pasadena, CA 91125, USA

Introduction. The quantification of the BOLD effect in vivo requires a detailed understanding of signal dephasing mechanisms of the microvascular network in macroscopically homogeneous tissue compartments. Following pioneering works (1, 2) substantial progress was achieved within Monte Carlo simulation (3, 4) and analytical models (5–7). In analytical models, the blood vessels are treated as magnetically homogeneous cylinders which is a good approximation for veins and venules (7), but is too simplistic for capillaries. A more realistic model in which capillaries are described as aligned spheres was considered only in Monte Carlo simulation (4). As a first step in analytical modeling, one needs a description of the dephasing effect of single spheres. One more problem for which the effect of spheres is relevant is the susceptibility effect of contrast agents (e.g. to quantify the signal changes during bolus passage).

Here we present an analytical model of the MR signal dephasing in the medium where the magnetic field is distorted by stochastically distributed spherical particles. We consider only the diffusion narrowing regime which is more relevant for small particles. The result has the form of a closed analytical formula. The parameters of our model are: B_0, the main magnetic field; χ, the magnetic susceptibility of particles relative to the surrounding solvent; D, the diffusion coefficient in the solvent; $\zeta(R)$, the differential volume fraction of particles with radius R which is normalized by the total volume fraction ζ_0: $\int \zeta(R)dR = \zeta_0$; and T_2 due to the spin-spin interaction. There are no adjustable parameters in our model.

Theory. Let us consider $S(t)$, the MR signal normalized to its value at initial time $t = 0$ immediately after RF excitation. For the low volume fraction of spheres, $S(t)$ can be expressed as follows (5–7)

$$S(t) = \exp\left[-\int_0^\infty f\left(\delta\omega\, t, \lambda\right)\zeta(R)dR\right]\exp\left(-\frac{t}{T_2}\right), \qquad (1)$$

where $\delta\omega = 4\pi\chi\omega_0$ with ω_0 being the cyclic Larmor frequency and $\lambda = D/(\delta\omega\, R^2)$. The f-function characterizes the strength of dephasing by a single sphere such that $(4\pi R^3/3)f$ is the effective volume in which the dephasing takes place. The f-function can be found from the general solution of the Bloch – Torrey equation (8). The dephasing regime considered here corresponds to the perturbative solution of this equation for the case $\lambda \gg 1$.

Results. We have obtained the complete signal time courses $S(t)$ for both the free induction decay (FID) and spin echo (SE) experiments. The f-function in Eq.[1] is dominated by its second-order term: $f = (2/15)\pi\lambda^2 F$ where

$$F = \begin{cases} a + G(a) & \text{for the FID} \\ a + 2G(a_0) + 2G(a - a_0) - G(a) & \text{for the SE} \end{cases} . \qquad (2)$$

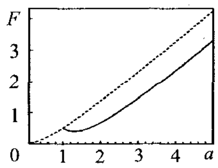
Here $a = \delta\omega\, t \cdot \lambda$, a_0 is the value of a for the time moment when the refocusing pulse is applied, and

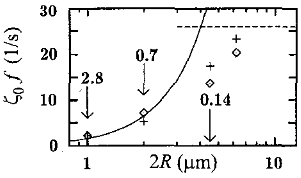
$$G(a) = \frac{a^3}{30} - \frac{11a}{15} - \left(\frac{a^5}{60} + \frac{a^3}{3}\right)\ln\left(\frac{\sqrt{a^2+4}}{a}\right) + \left(\frac{2a^2}{3} + \frac{8}{15}\right)\arctan\frac{2}{a} - \frac{4\pi}{15}. \qquad (3)$$

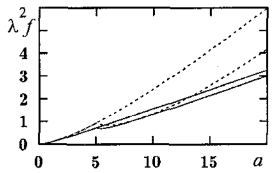
The function F is shown in Fig. 1 where the dashed and solid lines correspond to the signal time courses for the FID and the SE respectively. For large times, the difference between these functions is constant and equal to $8\pi/15$. The asymptotic forms of Eq.[2] for the FID are

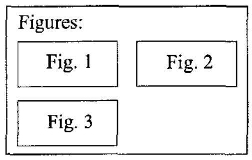
$$F(a) \approx \frac{\pi}{3}(\lambda\tau)^2, \quad a \ll 1\,; \quad F(a) \approx \lambda\tau - \frac{4\pi}{15}, \quad a \gg 1\,. \quad (4)$$

As a simple approximation to Eq[2] for the FID, one can join these asymptotic forms at $a = 1.2$.









Discussion. A verification of our model is shown in Fig. 2 in comparison with results of paper (3) for an almost monosized distribution of spheres for the FID. It shows the contribution to the relaxation rate from the dephasing effect of spherical particles as a function of particle diameter as found experimentally (diamonds), from the Monte Carlo simulation (crosses) and from our model (solid line). The horizontal line represents the limit for large R (5). The values of λ are shown for first experimental points. A good agreement between the results even for $\lambda = 0.7$ suggests that the validity range of our model $\lambda \gg 1$ can be practically understood as $\lambda > 1$.

It is interesting to compare the dephasing effect of spherical particles with that of cylinders found in (6). Such a comparison is shown in Fig. 3 in which $\lambda^2 f$ is plotted against a for spheres (solid lines) and for cylinders (dashed lines). The spherical particles result in a monoexponential signal decay at large times in contract to cylinders, for the case of which the logarithm of the signal is proportional to $a \ln a$ (6). This logarithmic factor enhances the dephasing effect of cylinders for large times as compared with that of spheres.

In its present form, our model can be applied to dilute solutions of small spherical particle of contrast agents. Applications to the discription of capillaries require solving a model with the alignement of spheres along the vessel. This work as well as the results for the static dephasing regime will be reported elsewhere.

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