Spiral SENSE: Sensitivity Encoding with arbitrary K-space Trajectories

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Introduction
Recently, sensitivity encoding with coil arrays has been discussed as a means of reducing acquisition time in standard Fourier imaging. The SENSE concept (1) combines reduced gradient encoding with the consideration of spatially distinct receiver sensitivities for signal localization. With rectilinear sampling patterns in k-space image reconstruction from sensitivity encoded data may be readily viewed as an unfolding process. This is due to the relative mildness of the undersampling effect in the rectilinear case, coupling only small numbers of pixels within disjoint cliques. However, for general patterns of undersampling k-space the unfolding approach does not hold. In particular, spiral read-out trajectories (2), being a popular concept in rapid imaging, have not been available in combination with sensitivity encoding.

In this work we present concepts for image reconstruction from sensitivity encoded data obtained along arbitrary sampling paths in k-space.

Methods
Image reconstruction is regarded as the action of a reconstruction matrix \( F \) upon the vector \( m \) of all sample values, one value from each receiver at each position in k-space. The resulting image vector \( i \) lists the pixel values to be generated.

\[
i = F m.
\]

The main goal in the choice of the reconstruction matrix is spatial resolution, i.e., the spatial weighting of signal reflected by a pixel value should be sharply centered about the center of the voxel it belongs to. Two concepts for designing \( F \) accordingly have been adopted from the sinc-shaped weighting functions obtained in standard Fourier imaging. Similar to those one may require that the weighting functions be best approximations to Dirac distributions in terms of least square deviation, yielding

\[
F = E^H C^{-1},
\]

where

\[
E_{(\gamma,K),P} = e^{i P (r)},
\]

\[
C_{(\gamma,K),P} = \int_{\text{vol}} e^{ik \gamma X(r)} e^{i P(r)} \, dr,
\]

where \( r \) denotes position, \( \gamma, K, P \) count the coils, positions in k-space, and voxels to be resolved, respectively, and the encoding functions are composed of gradient and sensitivity encoding.

\[
e_{(\gamma,K),P} = e^{ik \gamma X(r)} s_{(\gamma,P)}.
\]

Formula [2] entirely determines reconstruction, it may therefore be referred to as a strong voxel condition. A weaker condition is obtained by requiring that the weighting functions share another property of standard case sines, namely the fact that each is equal to one, at its own voxel center and zero at all others, or formally \( FE = \text{Identity} \).

If the number of coils times the number of k-space positions is larger than the number of pixels to be resolved, this condition leaves degrees of freedom to optimize image SNR. Noise is then voxelwise minimized by choosing

\[
F = (E^H \Psi^{-1} E)^{-1} E^H \Psi^{-1},
\]

where \( \Psi \) is the noise variance matrix reflecting noise levels and correlation of the receiver array used. Both formulae for \( F \) involve inversion of a rather large matrix. Formula [7] is weaker in terms of ensuring voxel quality. However, it is more convenient to use as it does not require the matrix \( C \). For preliminary studies formula [7] has been implemented on a DEC Alpha workstation. For minimizing calculation times the inversion step is carried out in k-space, that is, prior to inversion the matrix in brackets undergoes unitary transform using a Fourier matrix equivalent to FFT. Thus the number of significant matrix entries is reduced. Subsequent thresholding reduces computation time and memory requirements.

Results
Fig. 1a shows a 64x64 image of a quality phantom (Ø 200 mm), obtained with four surface coils surrounding the object and a full spiral sampling trajectory as displayed next to the image. The image in Fig. 1b was reconstructed from data undersampled with a spiral of just half the density. Besides a slight decrease in SNR, most prominent in the center, no loss of image quality is observed. Image reconstruction in both cases took roughly twelve minutes. Elimination of small entries after FFT was done row by row using 1% of the row modulus maximum as threshold. The sensitivity of image quality to this parameter is demonstrated by Fig. 1c), showing an image obtained with the reduced spiral and a threshold of 5%. The decrease in computation time by about 40% is embittered by considerable image defects.

Discussion and Conclusion
Spiral and any other non-rectilinear trajectories can apparently be successfully combined with and substantially be speeded up by sensitivity encoding. The weak voxel condition has been found sufficiently robust for well-behaved voxel coupling. Potentially, for treating near-singular systems the stronger condition may prove superior. Finally, while computation times presently are inconvenient increases in CPU power and numerical optimization may make the approach practical at some point in the future.

References