MRI Elastography Reconstruction Using A Harmonic Elastodynamic Model

Elijah Van Houten, Michael I. Miga, Francis E. Kennedy, John B. Weaver, Keith D. Paulsen
*Thayer School of Engineering, Dartmouth College, HB8000, Hanover, NH 03755

Abstract
Recently, imaging modalites such as ultrasound and magentic resonance (MR) have been used to measure subsurface displacements in tissue and several inversion schemes have been proposed to solve for stiffness properties. We have developed a finite element based inversion scheme which operates in a sweeping fashion on small overlapping subzones of the tissue space. The zone approach allows for a high degree of spatial discretization while maintaining algorithm convergence. Additionally, we are using a harmonic elastodynamic tissue model as the basis of our inversion and have shown accurate reconstruction simulations with up to 15% added noise.

Introduction
Palpation, although effective at diagnosing large near-surface cancerous tissue, is not an adequate technique for detecting small deep tumors. However, based on the success of palpation, the high contrast in stiffness between healthy and cancerous tissues remains an impetus for developing an elastographic imaging modality. MR and ultrasound elastography are the first imaging modalities to provide subsurface displacement data, which consequently provides tissue strain information, that can be used in an inverse method to recover stiffness properties.

Methods
Previous work has focused on estimating stiffness properties by calculating local wavelengths resulting from shear excitation of the tissue [1]. More recently model based reconstruction has been used in this same context [2]. It has been our experience that shear wave excitation of tissue is limited due to attenuation. In our approach, we use longitudinal harmonic waves which ultimately generate standing shear waves deep within the tissue.

The equations describing the elastodynamic response of soft tissue under an applied harmonic deformation are,

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \tau. \qquad (1)$$

where τ is the stress tensor, $\mathbf{u}$ is the displacement vector and ρ is the tissue density. Assuming that the material is excited harmonically at frequency ω, these equations can be solved in the frequency domain,

$$\rho \omega^2 \mathbf{U} = \nabla \cdot T. \qquad (2)$$

where $\mathbf{u} = \mathbf{U}e^{i\omega t}$. Depending on the constitutive relationships assumed between stress and strain, the presence of damping can be incorporated; however, for this discussion we have assumed Hookean dependence with a constant Poisson's ratio of $\nu = 0.49$; thus leaving Young's modulus the only unknown in the domain (recall that displacement data, $\mathbf{u}$, is measured from the imaging technique).

The inversion problem is a nonlinear Newton-based iterative scheme which minimizes the square of the error between measured and model-predicted values for each zone and solves for the distribution of Young's modulus. The zone domain is radially shaped and determined by a hierarchial ordering of local residual errors which cover the entire mesh (zone boundary conditions are determined from the MR dataset). After all areas of the mesh have been iterated on a specified number of times (zone iterations vary due to overlapping capability), a global forward problem is executed using the updated modulus distribution and the zone process begins again. The main advantage of this technique is that it allows sufficient discretization to resolve the wavelengths found in soft tissue harmonic motion.

Results & Discussion
Using this inversion scheme, a simulation was performed on a breast cross-section with complex phantom shapes of varying contrast. Random noise (up to 15 % of original displacement) was added to the forward solution data to simulate signal degradation in the MR measurements. This noisy solution is then operated on by the inversion algorithm described previously. Figure 1 shows the results with an initial guess of a uniform Young's modulus of 7000 Pa. The inversion process

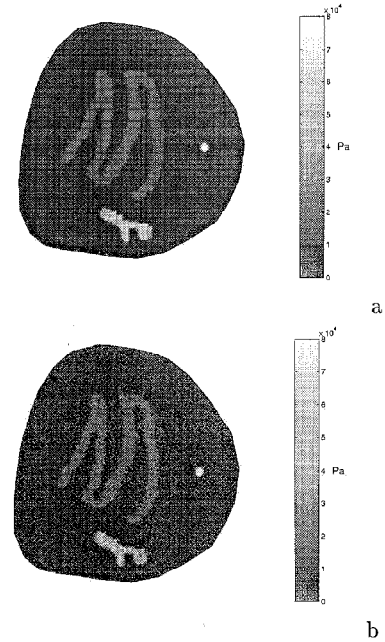

Figure 1: Breast computational phantom/reconstruction with regions of varying contrast (2x, 5x, 10x): (a) phantom Young's modulus distribution, (b) inverse solution with 15% random noise added to data.

consisted of 18 sweeps over the entire space, each sweep using roughly 1000 zones of about 150 elements and 100 nodes to insure that every node within the discretization was operated on at least once. Overall the results shown in our simulations are extremely encouraging. This work was supported by NIH grant R01-NS33900 awarded by the NINDS.

References
1. R. Mathupillai, et al. *Magnetic Resonance in Medicine*, 36, 266-274, 1996.
2. A. Manduca, et al. *Lecture Notes in Computer Science*, 1496, 606-613, 1998.