Enhancement of Anisotropic Diffusive Filtering of MR Images Using Approximate Entropy

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Introduction

Anisotropic diffusive filtering [1] provides an elegant method for reducing noise levels in digital images, whilst retaining much important image information. The original formulation of anisotropic diffusion and subsequent developments rely on measurements of local image structure to determine the degree of local diffusive smoothing to be applied. The presence of intensity gradients within an image is used to determine edges of objects that are to be preserved (or even enhanced) in preference to less significant intensity gradients, which are presumed to represent image noise or structures of negligible interest.

An alternative method for determining significant information in an image may be to determine whether the local spatial distribution of signal intensity is in some sense ordered, or random. We have investigated the use of an entropy-based statistic, approximate entropy (ApEn) (e.g. [2,3]) to determine local pixel intensity regularity. ApEn is a computable measure of sequential irregularity that is applicable to sequences of numbers of finite length [3]. As such, it may be used to determine how random a sequence of numbers is, even when the length of the sequence is short. We have previously demonstrated the applicability of ApEn to noise reduction with the creation of a modified median filter [4]. Here we present the use of ApEn to construct a modified anisotropic diffusion filter. We show that the ApEn modification leads to enhanced information retention in the presence of image smoothing, and show an example of an image smoothed using the modified anisotropic diffusion filter.

Methods

ApEn may be interpreted as a measure of the maximum frequency at which number sequences within a number series u of length m occur compared with sequences of length m+1. High values of ApEn imply a high degree of randomness; low values imply a high degree of order. Intuitively, ApEn may be thought of as a measure of the predictability of a sequence of numbers. For example, in the binary series ‘010101010101...’, it is clear that each ‘0’ will be followed by ‘1’, and vice-versa. Similarly, each ‘01’ will be followed by another ‘01’ (and each ‘10’ by another ‘10’). Such a sequence has high predictability, and has a low ApEn value. A sequence such as ‘0111101010000...’ is far less predictable, and has a finite high value of ApEn (see [2,3] for more information on approximate entropy).

We modified the standard anisotropic diffusion conductancy, \( g(\nabla I) \) [1], to include a function of ApEn, producing a new conductancy, \( G(\nabla I) \):

\[
G(\nabla I) = \exp \left( -\frac{1}{k} \right) \text{ApEn}
\]

where \( I \) is the image intensity, and \( k \) is the Canny ‘noise estimator’ [5]. Thus, the amount of smoothing applied is now dependent upon both the local image intensity gradient and the local intensity regularity, as measured by ApEn.

We compared the effect of the ApEn-modified filter with an unmodified anisotropic diffusion filter by comparing the overall image information retention for a given change in image noise. Noise levels were estimated by taking the mean standard deviation (SD) of the signal intensity within two regions of interest – one in the image background, and one in a region of uniform mean signal intensity from an anatomical region. Gaussian noise was added to the original image to provide a test ‘noisy’ image, and a ‘gold-standard’ image. Successive iterations of both the modified and unmodified anisotropic diffusion filters were then applied. After each iteration, noise levels were measured. The correlation coefficients between the processed images and the original image (without added noise) were measured to determine the degree of image similarity, and to provide a measure of information retention [6].

Results & Discussion

Figure 1 shows a region of a coronal brain image of a patient with multiple sclerosis before and after processing with the ApEn-modified anisotropic diffusion filter. After three iterations of the filter, the random noise in the original image has been effectively removed, with minimal effect on the anatomical information represented. This information retention is demonstrated in fig. 2, which shows how the information content of an image (as determined using the correlation coefficient, CC) varies after processing with the ApEn-modified filter. A perfect match between an image and the ‘gold-standard’ image would result in CC = 1. An optimum match is apparent for different noise levels for the two filters compared, showing that the ApEn modification allows more information to be retained, while achieving more noise reduction (i.e. a lower SD).

Figure 1. Detail of a coronal T1-weighted brain image before and after filtering with the ApEn-modified anisotropic diffusion filter (3 iterations).

Figure 2. Variation in CC as a function of noise level (expressed as mean SD). Crosses show ApEn-modified and squares show unmodified anisotropic diffusion filtering.

References


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