# A New Sampling Theorem for FT MRI of Multiple Regions

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## Introduction

We have generalized the conventional Fourier sampling theorem to cases in which the image vanishes except in multiple, non-adjacent regions within the field of view (FOV). By using this new "multiple region FT" (mrFT) sampling theorem, such images can be reconstructed from a fraction of the k-space samples required to image the entire FOV according to the conventional Nyquist sampling theorem. We have further applied this technique to images which have slowly varying signal across the entire FOV and have their high spatial frequencies (i.e. edges) confined to multiple separate regions. This method is quite different from the Locally Focused (LF) MRI technique [1], which takes advantage of the same prior knowledge. Because optimal k-space sampling is used, the mrFT method does not cause noise amplification, unlike LF MRI. Furthermore, it is no more computationally demanding than a simple Fourier transform. This technique should have utility in imaging objects which have their "edge" content confined to multiple small portions of the FOV. For example, the mrFT should be a useful alternative to conventional FT imaging in time-limited applications of MR angiography (MRA), such as first-pass Gd-enhanced 3D MRA of the carotid arteries. We believe that the greater efficiency of the mrFT sampling technique can be parlayed into increased temporal or spatial resolution -- both of which should improve the quality of these exams.

## Theory

The FOV is first divided into a uniform mosaic of rectangular "cells." The k-space representation of the image is sampled on a sparse grid, which may be offset slightly from the origin of k-space and which has spacing appropriate for FT imaging of a single cell. An inverse FT of this sparse data produces a "reduced FOV image" in which each cell contains both the actual contents of that cell as well as aliased copies of the contents of all other cells. Specifically, each cell in the "reduced FOV image" contains a linear combination of the contents of all cells, the weighting of each being a phase factor determined both by the relative location of the aliased cell and the k-space offset used in the sparse sampling pattern. If only $N$ cells of the original image contain non-zero signal, this relationship produces a linear equation in $N$ unknowns (each unknown being the imaged contents of one of the cells). If the image's k-space representation is sampled on $N$ sparse grids, each with a different offset, these can be used to generate $N$ linear equations with $N$ unknowns. Therefore, the image can be reconstructed simply by inverting an $N$x$N$ matrix in order to solve these equations. If the actual k-space offsets are chosen wisely, this matrix is unitary. Then, image reconstruction is perfectly well-conditioned (just as it is for a conventional FT), and there is no noise amplification during the reconstruction process. These optimal k-space sampling patterns can be determined analytically for simple arrangements of signal-containing cells. For example, when the non-zero cells are adjacent and form a larger rectangle, the optimal k-space sampling pattern for the mrFT reduces to the sampling pattern dictated by the conventional sampling theorem for a reduced FOV. In this report, we illustrate the method by finding and using the optimal sampling patterns for images that vanish except in two non-adjacent cells. The generalization to more than two intensity-containing cells is straight-forward.

## Methods

We applied the mrFT technique to two simulated objects. The 512x512 image of the first "object" was divided into an 8x8 array of cells that had vanishing signal in all but two of the cells. Simulated k-space data was generated by Fourier transforming this image. Only 1/32 of these data (i.e. two sparse sampling patterns) were used to reconstruct the image according to the mrFT method. A second simulated object was created by adding a smoothly undulating background throughout the FOV (Figure 1). This image was treated as the sum of a low-frequency "background" image $B(x,y)$ and an "edge" image $E(x,y)$. The background image was reconstructed from appropriately filtered data on a dense grid at the center of k-space using the conventional FT method. The complementary image $E(x,y)$ was essentially zero except in two cells and was reconstructed using two sparse sampling patterns as dictated by the mrFT technique. The complete image was then obtained by adding the reconstruction of $B(x,y)$ to the reconstruction of $E(x,y)$. The combined sampling pattern is shown in Figure 2, with a portion of the peripheral sampling pattern magnified to show the non-uniform spacing of the two sparse grids. Notice that the sampled data included only 3/64 of the k-space points (i.e. ~1/20 NEX) required to reconstruct a conventional FT image with 512x512 resolution.

## Results

The mrFT reconstruction of the first simulated image is not shown because it was exact to within computer round-off error. The mrFT reconstruction of the second simulated image is shown in Figure 3 and should be compared to the "exact" image in Figure 1. While no errors are noticeable in Fig. 3, they are apparent in Figure 4, which is the difference between Figs. 1 and 3, windowed to show low intensities. This error is caused by some low spatial frequencies that were not correctly represented in $B(x,y)$. These produce non-zero intensity in the complementary "edge" image outside the two cells and lead to aliasing when $E(x,y)$ is reconstructed by the mrFT. While the greatest error was approximately 8%, it is significant that this error is comprised solely of low frequency image components -- the edges are reconstructed faithfully. If the important structures *are* the edges (as in MR angiography), this error becomes even less important.

## Discussion

The mrFT reconstruction method offers significant advantages in imaging situations in which high spatial frequencies are present only in small regions of the FOV (e.g. MR angiography). The LF MRI method [1] takes advantage of the same prior knowledge, but differs from mrFT in fundamental ways. First, LF MRI can be computationally demanding because it uses a non-Fourier set of basis functions to estimate the missing Fourier components on a dense k-space grid. In contrast, image reconstruction by mrFT is even faster than a conventional FT of the usual dense k-space sampling pattern. This is because mrFT calculates the image *directly* from a small number of FTs of sparse data, using a unitary transformation which has small dimensions (i.e. the number of non-zero cells). Second, LF MRI tends to suffer from noise amplification due to ill-conditioning of the image reconstruction process, which involves a non-unitary transformation of k-space data. In order to minimize this noise amplification, it is necessary to oversample k-space, which reduces the efficiency of LF MRI. In contrast, the mrFT technique is *perfectly* conditioned because it involves only unitary transformations of the k-space data if the sparse sampling patterns are chosen correctly. Therefore, the mrFT *does not amplify noise*, and oversampling is not necessary.

This technique can be applied to any imaging scenario in which the edge content of the image is confined to small, non-adjacent regions of the FOV. We are currently working to apply it to 3D Gd-enhanced angiography for rapid, high resolution imaging of the carotid and vertebral arteries.

## References

1. Yao, L., Cao, Y., Levin, D.N., *Magn. Reson. Med.* 36, 834, 1996 and 37, 251, 1997.

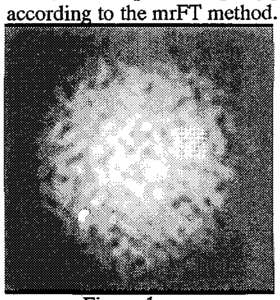
Figure 1

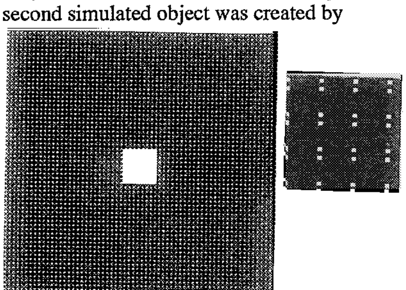
Figure 2

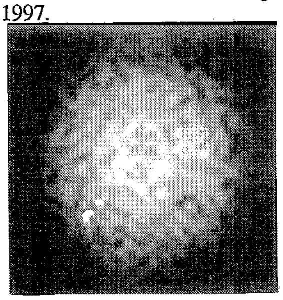
Figure 3

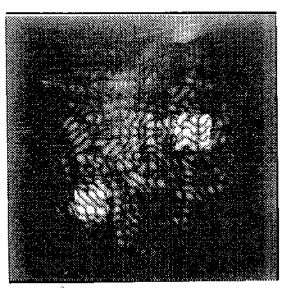
Figure 4